

**YOUNG
MASTER**



ONLINE ROUND

MATHEMATICS CHALLENGE

AGE CATEGORY: BUMBLEBEE (16-18 YEARS OLD)



Maths Topics for Bumblebee: 16-18 Years Old

Mathematics Knowledge Test will assess the following topics:

Algebra: Understanding and using algebraic concepts and notation, including equations, expressions, formulae, and functions, and solving equations and inequalities. This includes topics such as simplifying expressions, solving linear and quadratic equations, and graphing linear and quadratic functions. Also, it includes advanced algebraic concepts such as logarithms, matrices, exponentials and complex numbers.

Calculus: Understanding and using concepts of calculus such as limits, derivatives, integration, and solving differential equations. This includes topics such as differentiation and integration techniques, applications of calculus in optimization, related rates, and solving differential equations.

Trigonometry: Understanding and using concepts of trigonometry such as right triangle trigonometry, the unit circle and trigonometric functions. This includes topics such as solving problems involving trigonometric ratios and angles, and understanding the relationship between trigonometry and geometry.

Vectors: Understanding and using vector algebra and geometry, including vector addition and subtraction, scalar multiplication and vector products.

Series: Understanding and using concepts of series, including arithmetic, geometric and binomial series, series of functions and Taylor series.

Coordinate geometry: Understanding and using concepts of coordinate geometry such as graphing points on the coordinate plane, using coordinates to prove geometric theorems and polar coordinates.

Number theory: Understanding and using concepts of number systems, number theory and number properties, including prime numbers, factors, multiples and primes.

Proofs and mechanics: Understanding and using concepts of proof, including mathematical reasoning, mathematical induction, and mechanics.

1. The roots of the equation $x^2 - 7x + p = 0$ differ by 3.
What is the value of p ?

A. 21
B. 10
C. -21
D. -10

2. The expression $x^2 - 8x + 11$ is written in the form of $(x + h)^2 + k$, where h and k are real constants.

What is the value of k ?

A. -8
B. -5
C. -4
D. 4

3. How many real roots does the equation $x^4 - 8x^2 - 9 = 0$ have?

A. 0
B. 1
C. 2
D. 3

4. Given that $(x^2 + 2x + 3)(y^2 - 4y + 7) = 6$, where x and y are real numbers. What is the value of xy ?

- A. -2
- B. -1
- C. 7
- D. 14

5. The mid-point of line segment joining the points $A(a, -1)$ and $B(2, 3)$ is $M(b, a)$. What is the value of $a+b$?

- A. -1
- B. 0
- C. 1.5
- D. 2.5

6. ABC is a triangle with vertices $A(1, 2)$, $B(5, 2)$ and $C(9, 6)$. What is the area of triangle ABC ?

- A. 10
- B. 9
- C. 8
- D. 7

7. A circle has an equation: $(x - 1)^2 + (y - 2)^2 = 25$. What is the equation of the tangent to the circle at A(4,6)?

- A. $4y + 3x - 36 = 0$
- B. $3y + 4x - 36 = 0$
- C. $3y + 4x + 36 = 0$
- D. $3y - 4x + 36 = 0$

8. Which one is a possible value of k if the line $y = kx + 2$ is tangent to the circle $(x - 5)^2 + (y - 2)^2 = 20$?

- A. -4
- B. -3
- C. 1
- D. 2

9. Let $0^\circ \leq x \leq 90^\circ$ and $\sin x = 0.8$. What is the value of $\tan x$?

- A. 0.6
- B. -0.6
- C. 0.75
- D. $\frac{4}{3}$

10. How many solutions does the equation $\sin 2x = \cos x$ have?
($0 \leq x \leq 2\pi$)

- A. 0
- B. 1
- C. 4
- D. 2

11. Let $f(x) = 5 \sin x + 12 \cos x$, where $x \in \mathbb{R}$. What is the maximum value of $f(x)$?

- A. 17
- B. 13
- C. 7
- D. -7

12. What is the constant term in the expansion of $\left(x + \frac{2}{x^2}\right)^6$?

- A. 240
- B. 120
- C. 60
- D. 30

13. The sum of the first n terms of a series is given by

$$S_n = n^2 + 3n. \text{ What is the } n\text{th term of the sequence?}$$

- A. $n + 3$
- B. $2n + 1$
- C. $2n + 2$
- D. $2n + 3$

14. Let a_n be an arithmetic progression with $a_1 = 2$ and common difference 3. What is the sum of the first 100 terms?

- A. 15,050
- B. 10,050
- C. 7,550
- D. 5,050

15. Let a , b , and c be positive integers, where $a \geq b \geq c$, satisfying

$$a^2 - b^2 + c^2 - ab = -2017 \text{ and}$$

$$a^2 + 3b^2 + c^2 - ab - 2ac - 2bc = 2023$$

What is the value of a ?

- A. 2018
- B. 2019
- C. 2020
- D. 2021

16. Given that x , $x + 3$ and $3x + 3$ are the first three terms of an increasing geometric sequence. What is the sum of the first 10 terms?

- A. 6141
- B. 3069
- C. 1533
- D. 1299

17. Let $f(x) = x^3 + ax + b$, where a and b are constants. $f(x)$ has a turning point at $(2,6)$. What is the value of $a+b$?

- A. 34
- B. 22
- C. 12
- D. 10

18. How many prime numbers p are there such that $p+2$ and $p+4$ are also primes?

- A. Infinitely many
- B. More than 1000 but finite
- C. 4
- D. 1

19. How many triples (x, y, z) of positive integers are there satisfying $x + y + z = 11$?

(Triples $(1, 1, 9)$ and $(1, 9, 1)$ count as different.)

- A. 10
- B. 15
- C. 45
- D. 55

20. Let x and y be positive integers.

How many pairs of (x, y) satisfy the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{5}$?

- A. 0
- B. 1
- C. 2
- D. 3

21. The curve C has equation $y = 3x^4 - 8x^3 - 3$

Which value of x verifies a stationary point $(\frac{dy}{dx} = 0)$?

- A. 1
- B. 2
- C. 3
- D. 4

22. $f(x) = 2x^3 - 5x^2 + ax + a$

Given that $(x + 2)$ is a factor of $f(x)$,
find the value of the constant a .

- A. 12
- B. -12
- C. 36
- D. -36

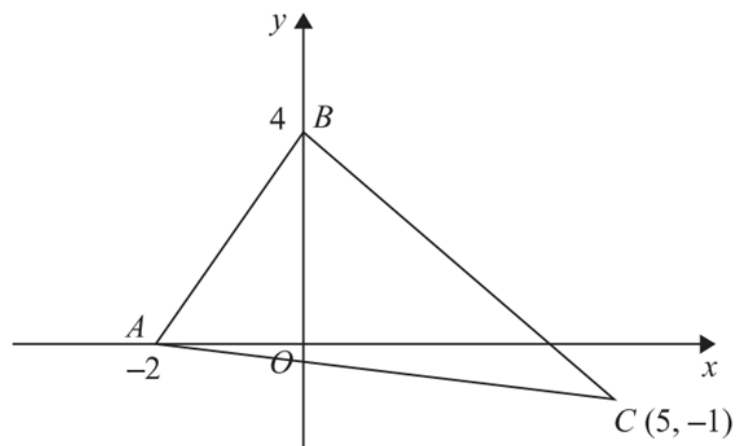
23. Given $y = x(2x + 1)^4$ and $\frac{dy}{dx} = (2x + 1)^n(Ax + B)$

Where n , A and B are constants, which value below is not correct?

- A. $n = 3$
- B. $A = 10$
- C. $B = 1$
- D. $A = 8$

24. What is an equation of the line that passes through C and perpendicular to AB ?

- A. $y = -2x + \frac{2}{3}$
- B. $y = -x + \frac{1}{3}$
- C. $y = -\frac{1}{2}x + \frac{2}{3}$
- D. $y = \frac{1}{2}x - \frac{2}{3}$



25. There are 10 pens in a box. There are x red pens and all the other pens are blue.

Jack takes at random two pens from the box.

Find an expression, in terms of x , for the probability that Jack takes one pen of each colour. Give your answer in its simplest form.

A. $\frac{10x-x^2}{45}$

B. $\frac{10x-x^2}{90}$

C. $\frac{20x-2x^2}{45}$

D. $\frac{x^2-10x}{45}$